Social security pension systems as public goods

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The study demonstrates that using the apparatus of repeated games by assuming methodical individualism the present social security pension systems are in multiple prisoner’s dilemma situation. To describe this multiple prisoner’s dilemma situation we shall formulate and demonstrate four statements, the demographic dilemma, dilemma of paying contributions and that of the political class. The dilemmas will be demonstrated through the so-called Selten’s theorem in the theory of repeated games.

Keywords: repetitive games, multiple prisoner’s dilemmas, public goods, social security pension systems.

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Introduction

By assuming methodological individualism and presuming that members of the society wish to optimise rationally their own profit in some form, in the following we shall demonstrate through using the apparatus of games theory Olson’s theorem, namely that public goods are not necessary produced in quantity optimal for the community. In the article we formulate four statements that express the necessity of the defects in present – publicly managed, defined benefit, pay as you go, actuarially unfair- social security pension systems.

1. Non-cooperative games in normal form

Let

\[ N = \{1, \ldots, n\} \]
\[ S = \{S_1 \times \ldots \times S_n\} \]
\[ u : S_1 \times \ldots \times S_n \rightarrow \mathbb{R}^n \]

denote the number of players

denote the strategic set

denote the pay-off function.

(1.1) DEFINITION: Let \( G \) be an \( n \) players game in normal form:
\[ G = \{N, S, u\} \]

thus the certain profit of player \( i \) is:
\[ \alpha_i = \sup_{s_i \in S_i} \inf_{s_{-i}} u_i(s_i, s_{-i}), \quad \text{ahol} \quad S_{-i} = S_1 \times \ldots \times S_{i-1} \times S_{i+1} \times \ldots \times S_n, \]

(1.2) DEFINITION: \( s_i^* \in S_i \) is prudent strategy of player \( i \) exactly if
\[ \inf_{s_{-i} \in S_{-i}} u_i(s_i^*, s_{-i}) = \alpha_i. \]

(1.3) DEFINITION: We call game \( G \) inessential if \((\alpha_1, \ldots, \alpha_n)\) pay-off does not dominate in Pareto sense, namely

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(1.4) DEFINITION: Nash equilibrium:

We define the Nash equilibrium of a strategic game as follows: \( G(N, S, (\pm)) \) is a \( s^* \in S \) Nash equilibrium if

\[
(s^*_i, s^*_j) \pm_i (s^*_i, s^*_j) \quad \forall s_i \in S_i
\]

applies to \( \forall i \).

(1.5) DEFINITION: The best response of player \( i \) to some \( s_{-i} \) strategy vector is \( BR_i(s_{-i}) \):

\[
BR_i(s_{-i}) = \{ s_i \in S_i : (s_i, s_{-i}) \pm (s_i, s_{-i}) \forall s_i \in S_i \}
\]

and we call \( BR \) the best response function of \( i \).

Note: using the above notion, Nash equilibrium is a strategy vector \( s^* \) to what applies:

\[
s^*_i \in BR_i(s^*_j) \quad \forall i \in N.
\]

(1.6) DEFINITION (Multiple Prisoner’s Dilemma):

1. \( \forall i \in N \) player has two strategies: \( S_i = \{ s^\text{cooperation}_i, s^\text{desertion}_i \} \),
2. \( \forall i \in N \) \( u_i(s_1, \ldots, s^d_i, \ldots, s_N) > u_i(s_1, \ldots, s^k_i, \ldots, s_N) \),
3. \( \forall i \in N \) \( u_i(s^k_i, \ldots, s^k_N) > u_i(s^d_i, \ldots, s^d_N) \).

(1.1) EXAMPLE: Let \( m \subset N \) \( s^k_i \quad \forall i \in \{ m \} \) sign be \( x = m/N \). Thus \( u_i(x) \) and \( u_j(x) \) are the pay-offs: \( u_i(x) > u_i(x) \) and \( u_i(1) > u_j(0) \).

1. Finitely Repeated Games

We call games that are played only once one shot games, and those that are played multiple times repeated games. The latter ones are often called super games, too.

In case of repeated games, we derive the strategy set of the new super game and their pay-off functions from the original game.

(1.1.1) THEOREM: Let \( G \) be a finitely repeated game. Let us suppose that the stage game has Nash equilibrium and it is unique. Thus the Nash equilibrium of \( G \) is subgame perfect, and contains the Nash equilibrium of the stage game.

Proof: through induction to the game period.

\( T = 1 \) repeated game is the elementary game that has unambiguous Nash equilibrium.

Let us suppose that the statement is true up till \( T - 1 \). Let us consider game repeated \( T \) times. Each partial game that starts in the second period contains a game repeated \( (T - 1) \) times that has a satisfactory Nash equilibrium. Thus the acts of the first period do not influence the next periods. Therefore the equilibrium of the first period consists of the best responses, which means that even during the first period the Nash equilibrium of the elementary game emerges.

(1.1.2) THEOREM (folklore): In case of all finitely repeated games, the repeated game has an
unique subgame perfect Nash equilibrium if the stage game has a unique Nash equilibrium.

Proof: through induction.

\( t = 1 \) the repeated game is the stage game, thus the statement is true because of the condition. Let us suppose that the statement is true to \( t - 1 \); in this case, all games commencing in each \( t \)-th period contain the game repeated \( t - 1 \) times that has unique equilibrium. This equilibrium is the best response in the basic game; therefore, this equilibrium satisfactory for the subgame has to be the equilibrium for the whole game.

(1.1.3) Theorem (Selten): If the finitely repeated game has unique equilibrium, it is the solution of the game in all periods.

2. Reputation

In case of repeated games, especially of prisoner’s dilemma games, the model of reputation is often used. The notion of reputation is usually used in two senses. First, reputation is gained thanks to our systematic choice of strategy during earlier phases of the game. As a result, other players draw conclusions as to our subsequent behaviour. Second, when playing a game we wish to meet norms out of game, thus sustaining our appreciation in the group. (That is to say, we diverge from the earlier concept of rational player in both cases.) In numerous articles, Fudenberg demonstrated (e.g. Fudenberg, 1991, 1998) that by the introduction of reputation the cooperation may arise also in case of finite prisoner’s dilemma game.

2. Public Goods

The standard handbooks are not unanimous in defining the notion: we read the followings in ‘Economy of The Public Sector’ by Joseph E. Stiglitz:

‘The pure public goods have two major attributions. First, their usage cannot be divided. Second, the division of their usage would not be advisable and practical.’

In their book ‘Public Finance and Public Choice’, John Cullis and Philip Jones write the following:

‘The public goods (or „collective goods” or „social goods”) are defined by Samuelson (1954, p. 387) as „such goods that are consumed communally, therefore their consumption does not lead to their less consumption by others.” He stresses that the public goods markedly differ from private goods that „can be divided among individuals” (Samuelson, 1954, p. 387). In case of private goods, the total consumption is the sum of individual consumptions.’

Non-rivalness in consumption

An attribution deriving from the definition of public goods is that their consumption is not competitive, that is to say, the consumption of an individual does not reduce the profits of all the others. As a consequence, if we wish to give account of the summarised demand of a certain good, we have to summarise vertically, and not horizontally, the individual demand curves.

(2.1.) Definition Non rivalness: If one use some of it, the good is still fully available to everyone.
Lack of exclusion

Second attribution of public goods is that the consumers cannot be excluded from the profits – or the exclusion would require a very expensive method. If the good is available, one individual cannot exclude the other from its consumption. In case of private goods, the market works so that consuming the good is only possible by paying its ‘price’. Anybody can be excluded from the consumption if they cannot get possession of the good.

(2.2) DEFINITION Non-excludability: If the good is available to anyone, it is available to all. It is impossible to exclude anyone.

According to the two aforesaid characterising attributes, the goods are usually classified (Blümel, Pethig, Hagen 1986) as follows:

<table>
<thead>
<tr>
<th>Excludability</th>
<th>Rivalness in consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
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<td></td>
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</tr>
</tbody>
</table>

In order to comprehend the behaviour of stowaway, we have to define the communal decisions. Elster (Elster 1978) defines the communal decision saying: ‘we understand communal decision as choice taken by all or almost all members of a given group and the result of the decision has effects on communal level’. The above definition leaves open the question of the conditions and motives the decision has been made by. Within communal decisions, Elster defines the group of decisions a commune takes before creating a public good.

Let us define the dilemma of communal decision as follows: Let $U$ be an interpersonal outcome structure (e.g. the game theory structure of the situation). In this case, we regard $U$ as a dilemma of communal decision exactly if there is contradiction between acts rational on individual and on communal levels.

Note: The prisoner’s dilemma games are perfect examples of this, when the equilibrium set by individually rational acts, the Nash equilibrium, and the best state for the commune, the Pareto optimum, do not coincide.

(2.2) DEFINITION: We regard member $A$ of commune $C$ as having stowaway behaviour in point of the action to produce the given public good exactly if:

1. The individual aims at exiting the production of public good.
2. Individual $A$ assumes that the efforts of $C$ members of the commune suffice for the production of the given public good.
3. Individual $A$ assumes that his or her participation in the production of the public good is necessary only if adequately informed members of the commune participate in the communal effort that will finally result in the practical production of the given public good.
4. Individual $A$ assumes that his or her individual profit of deserting the production of the public good is higher than that of participating in the production of the public good.
5. Individual A assumes that if all members of the commune participate in the production of the public good, its usefulness is higher for everyone than everyone’s desertion.

6. Individual A assumes that his or her absence from the production of the public good results in cost (that can be even nought) to the members of the commune participating in the production of the public good.

*Note:* The latter three criteria formulate the intention of stowaway behaviour, while the second and the third attempts at interpreting the desertion, and the first points to the whole decision situation.

The above definition does not suppose the given individuals being in a game theory situation, only that they are capable of measuring for themselves the rationality of participating in the production of the public good. The above definition, nevertheless, is well applicable to situations defined with game theory apparatus.

The stowaway behaviour is usually approached through prisoner’s dilemma, but we think it is not necessarily so, since the most important component for the individual of the situation is the conflict between individual interest and public good, or rather, that no cooperation or coordination between the given individuals does not exist. In the following, we shall examine the problems of communal decisions in game theory approach.

When the members of the commune carry out some common act, we presume that at least some members of the commune act in the interest of the production of the given good. The question is: who are they and what proportion of the good do they produce? In case of formal groups, such as organisations, some rules clarify who is or are to act. But in informal groups or in groups not well defined, it is less unequivocal. This is where the problems with communal decisions often arise (Olson 1965, Ostrom 1990).

(2.1) **THEOREM** (Olson): *The public goods are underprovided.*

### 3. Pension System as Public Good

We have dealt with the notion of public good in the previous chapter, and established that the exact definition is not uniformly accepted in the economical literature. Earlier we defined public good as a non-exclusive, non-competitive good. Using this economic terms in not without problems since our concept of public and private good has often changed in space and time. Let us consider that the public production of goods was less frequent in earlier times than nowadays. The public services we regard as public goods were not provided by the communes to their members, and very often services we now regard as private goods are considered public in numerous communes. Today the land, the woods and their animals, are private goods, whereas they were public in the majority of earlier societies.

(3.1) **DEFINITION:** We consider potential public goods those goods that can be turned to non-exclusive, or non-rival, or both.

(3.2) **DEFINITION:** We regard de facto public goods as ones that are non-exclusive and available for all to consume.

We shall use the above two notions in the following.

1. **Children as public goods**

Children are the potential resources for the present pay-as-you-go pension systems since the
contributions of the next generation makes up the source of the provision of the current elderly. The rate of social security contributions depends to a large extent upon the fecundity; that is to say, increase in birth gives increasing return to the pension systems. The Bismarck pension systems set the pensions according to the previous contributions (as we have done already earlier); whereas the Beveridge systems cut the amount of the basic pension adrift from contributions. Consequently, these systems allocate the same pensions to families having brought up more children as to ones having no children; that is to say, they separate the security of individuals and generations from the number of children brought up and the fact that they ever had children at all.

Several authors – (good review is given by, Cigno 1991) – argue that the pay-as-you-go pension systems are responsible for the decrease in fecundity after the Second World War. In the following we shall delineate two important sequences of thought. Model of Barro and Becker (Barro R.J., Becker G., S.: 1989), as well as that of Caldwell are usually considered the two most important approaches. The two models can be summarised as follows: The model of Barro and Becker regards the self-value of the child as motive of fecundity; whereas the model of Caldwell (Caldwell 1982) regards the parent as wanting to assure his or her security of old age. In our view, the latter describes better the reality and the analyses in the literature show that Caldwell’s model can easily be expressed in numbers and his approach depict a model fitting the data.

If we consider the next generation public good according to our sequence of thought previously described, we can adopt the theorems of Olson and Selten, so we can state that the decrease in fecundity is a necessary process and reproduction in the next generation necessary for equilibrium falls off.

(3.1) THEOREM: **By assuming pay-as-you-go pension systems and individuals wishing to maximise their security of old age and their consumption during their active career, decrease in fecundity is necessary.**

**Proof:** If the services provided by the pay-as-you-go system are independent from the number of children brought up by the individual, the individual can maximise his or her consumption by minimising the number of children and maximising his or her supply of work force. The next generation can be considered public good, because no-one can be excluded from the payments of old age services who is somehow entitled; in this sense, the pension services satisfy the at least the non-exclusivity aspect of the notion public good; and it can be seen as non-competitive in the sense that the extent of the services is set in laws, therefore it is not competitive at least in short run. Thus the conditions of using Olson and Selten’s theorems are fulfilled; that is to say, the public good will be under-provided, in this case, the fecundity is inevitably decreasing.

**Note:** A consequence of the consideration of the above theorem is that if we link pension payments to the scale of ‘effort in producing’ the next generations it may be possible according to the logic of our demonstration that the public good, the next generation in this case, is produced in sufficient number.

**Note:** The above mentioned thoughts obviously need further examination, since the fact that the conditions used during the demonstration of a theorem are not fulfilled and therefore the theorem cannot be demonstrated this way does not mean that other processes may not lead to the creation of the phenomenon.
2. Inclination to pay contribution in case of the unfair pay-as-you-go system

The pay-as-you-go systems of numerous countries have many contradictions. The link between the contribution of the individual and the later services is rather loose. The rules have been changed very often in the last decades, what has rendered this loose link more uncertain to the contributors. The situation can well be described by the fact that the contributions of the decisive majority of those who are not paid employee wages are assessed on the basis of minimum wages according to the contribution statistics, that is to say, all that can afford pay the minimal contribution. By our time the pension services have become public goods, hence avoiding or minimising of paying the contribution is a rational strategy to the contributors, if feasible.

(3.3) DEFINITION: The pay-as-you-go system can be actuarially correct, when the contributions and the benefit payments of a given individual are balanced, and it can be actuarially incorrect, when this equality doesn’t hold.

3.2. THEOREM: Assuming rational individuals, the best response strategy is to minimise contribution to the possible minimum if the link between services and contributions is degressive.

Proof: The contributors compare their contributions and their expected pension payments and they can find out without using discount factor that the rate of payment is uncertain, or else it will be less than their cumulative contribution. On the other hand, they can observe that the balance of contributions and future pensioner payments will be positive if they pay only the contribution required for the minimal retired pay. Consequently, the rational behaviour is the minimisation of the contribution payments.

Note: Taking the situation in Hungary as an instance, the average duration in old age pension among males is thirteen years with the present pension rules, whereas it is seventeen years among females. The amount of minimal retired pay is approximately 21,000 forints. According to the present regulations, twenty years of employment and contribution paid on the basis of minimum wages are necessary to get minimal retired pay. The latter is approximately 13,000 forints a month. After a short calculation, without using any discount factor, the difference is more than one million forints in case of females, and 136,000 forints in case of males; that is to say, paying contribution on the basis of the minimum wages is a profitable strategy for both sexes.

3. Dilemma of the political class

Before formulating our argument, it is worth analysing the below table.

(3.3.1) Table : Pension Rises in Hungary 1991-2003

<table>
<thead>
<tr>
<th>Year</th>
<th>Index of net average wage %</th>
<th>Index of Inflation %</th>
<th>Index of pension benefits %</th>
<th>Real value of pension %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>125.5</td>
<td>135.0</td>
<td>125.6</td>
<td>−7.0</td>
</tr>
<tr>
<td>1992</td>
<td>121.3</td>
<td>123.0</td>
<td>120.0</td>
<td>−2.4</td>
</tr>
<tr>
<td>1993</td>
<td>117.7</td>
<td>122.5</td>
<td>118.0</td>
<td>−3.7</td>
</tr>
<tr>
<td>1994</td>
<td>127.3</td>
<td>118.8</td>
<td>124.8</td>
<td>5.1</td>
</tr>
<tr>
<td>1995</td>
<td>112.6</td>
<td>128.2</td>
<td>115.4</td>
<td>−10.0</td>
</tr>
</tbody>
</table>
The above drawing shows the pension rises in the last fourteen years is Hungary. The course of the election cycles is clearly observable on the drawing. It is obvious that the political class attempts to buy votes through pension rises. In this sense, the political class perfectly suits Buchanan’s analysis and Tullock’s ‘rent-seeking’ notion.

The dilemma of the political classes can be summarised as follows: Is driving of pension rises off their equilibrium and thereby buying votes possible and worth trying? The above strategy is very devastating in social and economic senses (as already analysed), since it demonstrates to the populace that the rate of the pension depends on the political classes and not on the contributions (which is a mistake). The above situation is a typical repeated prisoner’s game dilemma, where cooperation means abstaining from irresponsible promises, while desertion means uttering such promises. A clear consequence of Selten’s theorem is that the solution in equilibrium will be the desertion; that is to say, the necessary occurrence of irresponsible promises overbidding the previous.

(3.3.1) THEOREM: In absence of set pension rules and in case of pensioners wishing to maximise their consumption, the political class necessarily issues irresponsible pension promises.

Proof: It follows from Selten’s theorem.

Note: In Hungary, the pension rules are less stable than in more fortunate countries, therefore such short run promises are not regularly issued in those countries.

All over the world, the pension systems need transformation. During the flourishing era of the 1960s and 1970s, these systems made impractical promises, or the assumptions that would have made those promises tenable changed (as we analysed in details in chapter 3). Therefore the pension promises are annulled all over Europe. It is, however, clearly obvious that the annulations take place in ad-hoc manner in most of the countries, and not along a plan. The schema of prisoner’s dilemma well explains this phenomenon, since the political party that annuls the pension promises can face spectacular loss in popularity, whereas the political force that does not can reach increase in popularity. Thus we are in a classical prisoner’s dilemma situation.

(3.3.2) THEOREM: As long as the voters are interested in maximising their income in the short run and the political class in maximising votes, the long run reforms will be delayed as long as the constraints enforce them.

Proof: It follows from Selten’s theorem.

Conclusions

Above we have formulated four statements that express the necessity of the defects in present
social security pension systems. Moreover, these defects conglomerate in a circle, intensifying the others, and thus result in a ‘negative’ autoregressive process. The behaviour of the political class lessen in the short run the discipline and inclination to pay contributions, the expectations of the voters inspire the political classes to do so; and in the long run, such behaviour of the political class renders the substantive reforms impossible and reduces the productivity. These two processes, in turn, undermine fundamentally the sustainability of the system.

References